Closing today: HW_6A, 6B
Closing Wed: HW_6C, 7A
Exam 2 is Thurs: 6.3, 6.4, 6.5, 7.1-7.5, 7.7, 7.8

7.7 Summary: Two new approx. methods
Here is an example for n = 4 subdivisions:

1. Compute $\Delta \boldsymbol{x} = \frac{\boldsymbol{b}-\boldsymbol{a}}{\boldsymbol{4}}$.
   Label the tick marks: $\boldsymbol{x_i} = \boldsymbol{a} + \boldsymbol{i}\Delta\boldsymbol{x}$
2. Use formula.

Entry Task: With n = 4, use both new methods to approximate (just set up)

$$\frac{1}{\sqrt{2\pi}}\int_{-1}^{1} e^{-\frac{1}{2}x^2}\,dx$$

$\Delta \boldsymbol{x} =$, $\boldsymbol{x_0} =$, $\boldsymbol{x_1} =$,
$\boldsymbol{x_2} =$, $\boldsymbol{x_3} =$, $\boldsymbol{x_4} =$.

Trapezoid rule:

$\frac{1}{2}\Delta x[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)]$
=

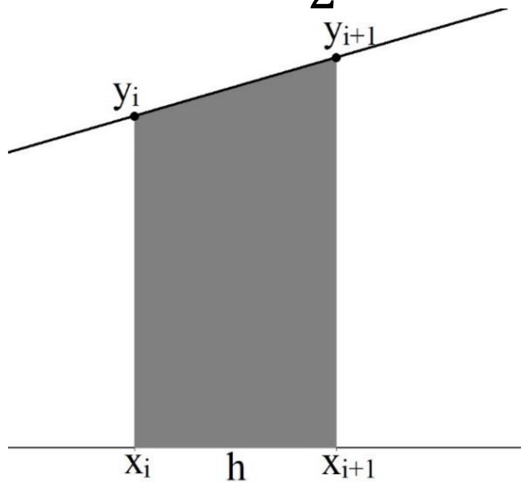
Simpson's rule:

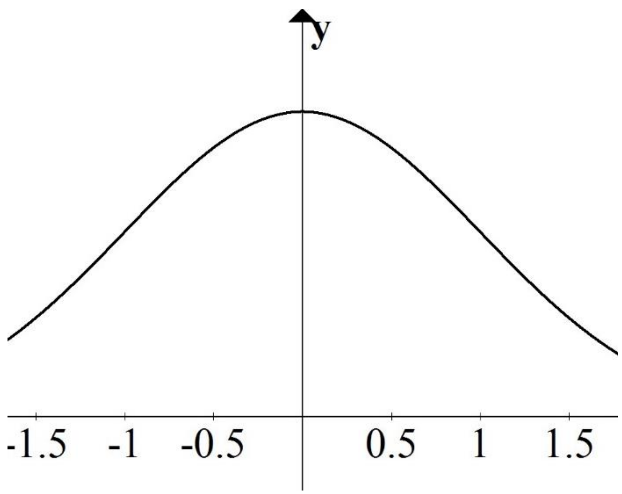
$\frac{1}{3}\Delta x[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)]$
=

7.7 Quick Derivation Notes:

Trapezoid Rule:

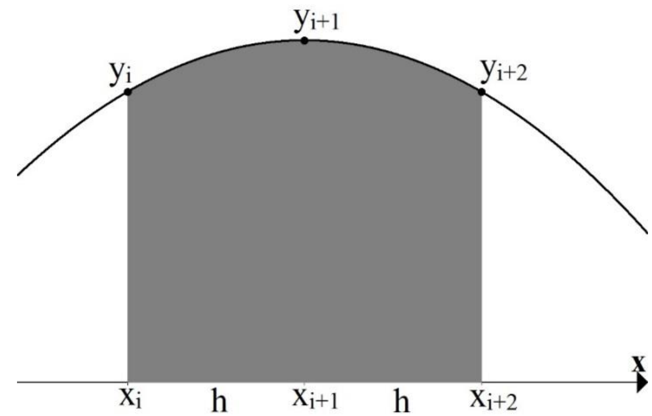
$$\text{Shaded Area } = \frac{h}{2}(y_i + y_{i+1})$$

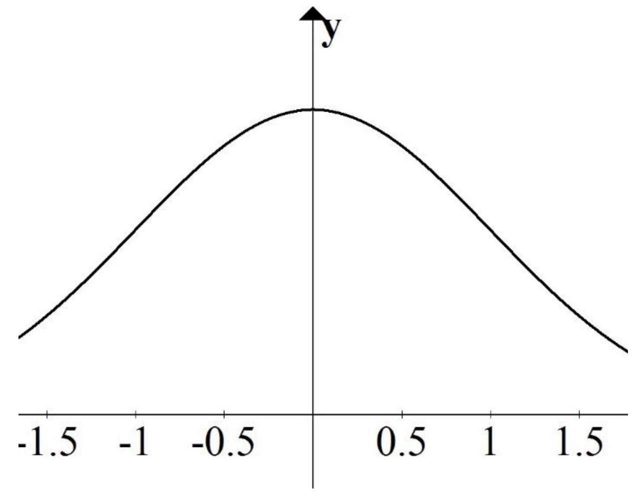




Simpson's Rule: If the curve below is a ***parabola*** ($y = ax^2 + bx + c$) that goes through the three indicated points, then

$$\text{Shaded Area} = \frac{h}{3}(y_i + 4y_{i+1} + y_{i+2})$$





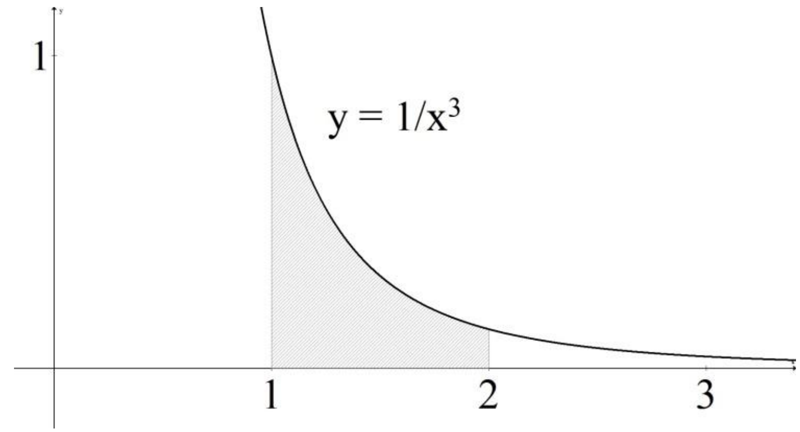
7.8 Improper Integrals

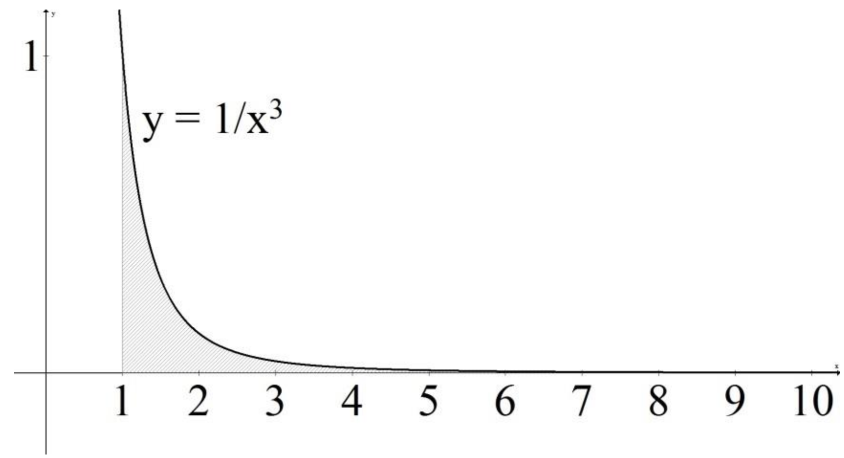
Motivation:

Consider the function $f(x) = \frac{1}{x^3}$.

Come the area under this function from…

1. x = 1 to x = t
2. x = 1 to x = 2
3. x = 1 to x = 100





Definition: (Book terms: infinite integral of integration, type 1 improper)

$$\int_a^\infty f(x)dx = \lim_{t\to\infty} \int_a^t f(x)dx$$

$$\int_{-\infty}^b f(x)dx = \lim_{t\to-\infty} \int_t^b f(x)dx$$

If the limit exists and is finite, then we say the integral *converges*. Otherwise, we say it *diverges*.

$$\int_{-\infty}^\infty f(x)dx = \lim_{r\to-\infty} \int_r^0 f(x)dx + \lim_{t\to\infty} \int_0^t f(x)dx$$

In this case, we say it *converges* only if both limits <u>separately</u> exist and are finite.

Example:

1. $\int_0^\infty \frac{1}{x^3}\,dx =$

Example:

2. $\int_{-1}^{\infty} e^{-2x} dx =$

3. $\int_{1}^{\infty} \frac{1}{x} dx =$

Definition: (Book terms: infinite discontinuity, type 2 improper)

If f(x) has a discontinuity at x = a, then

$$\int_a^b f(x)dx = \lim_{t\to a^+} \int_t^b f(x)dx$$

If f(x) has a discontinuity at x = b, then

$$\int_a^b f(x)dx = \lim_{t\to b^-} \int_a^t f(x)dx$$

If the limit exists and is finite, then we say the integral *converges*. Otherwise, we say it *diverges*.

If f(x) has a discontinuity at x = c which is strictly between a and b, then

$$\int_a^b f(x)dx = \lim_{r\to c^-} \int_a^r f(x)dx + \lim_{t\to c^+} \int_t^b f(x)dx$$

In this case, we say it *converges* only if both limits <u>separately</u> exist and are finite.

Example:

1. $\int_0^1 \frac{1}{\sqrt{x}} dx =$

Example:

2. $\int_{0}^{2} \frac{x}{x-2}\, dx =$

Limits Refresher

1. If stuck, plug in values "near" t.
2. Know your basic functions/values:

$$\lim_{t\to\infty} \frac{1}{t^a} = 0\,, \qquad \text{if } a > 0.$$

$$\lim_{t\to\infty} \frac{1}{e^{at}} = 0\,, \qquad \text{if } a > 0.$$

$$\lim_{t\to\infty} t^a = \infty\,, \qquad \text{if } a > 0.$$

$$\lim_{t\to\infty} \ln(t) = \infty.$$

$$\lim_{t\to 0^+} \ln(t) = -\infty.$$

3. For indeterminant forms, use algebra and/or L'Hopital's rule

Examples:

$$\lim_{t\to 1} \frac{t^2 + 2t - 3}{t - 1} =$$

$$\lim_{t\to\infty} \frac{\ln(t)}{t} =$$

$$\lim_{t\to\infty} t^2 e^{-3t} =$$

Aside:

A few general notes on **comparison**:

Suppose you have two functions f(x) and g(x) such that 0 ≤ g(x) ≤ f(x) for all values.

(a) If $\int_1^\infty f(x)dx$ converges,
then $\int_1^\infty g(x)dx$ converges.

(b) If $\int_1^\infty g(x)dx$ diverges,
then $\int_1^\infty f(x)dx$ diverges.

You can verify that

$$\int_1^\infty \frac{1}{x^p}dx\,, \qquad \text{converges for } p > 1.$$

$$\int_1^\infty e^{px}\,dx\,, \qquad \text{converges for } p < 0.$$

And you can compare off of these to sometimes quickly tell is something is converging or diverging (without calculating anything)